Constructive Accumulation: a look at self-organizing strategies for aggregating temporal events along the rhythm-timbre continuum

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ABSTRACT

This paper aims to summarize several algorithmic techniques taken from dynamical system modeling that can repurposed to produce 'constructive accumulation', a paradigm for describing the process by which concurrent, rhythmic events or sound sources gradually converge over time. Using different behaviors of self-organization from dynamical systems modeling, these strategies employ networks of locally and globally coupled oscillators that can generate a range of swarm-like acoustic textures, each reflecting a range of synchronous states along a rhythm-timbre spectrum. More specifically, I review the behavioral dynamics and clustering kinetics of 'scrambler' oscillators, pulse-coupled oscillators, and a group of Kuramoto oscillators with feedback. I also provide a basis for sonifying such models, briefly touching on concepts in psychoacoustics that are relevant to the design of such sonic systems. Lastly, I demonstrate the way in which they're implemented, controlled, and composed for as generative models in my recent artwork, 'In Praise of Idleness'.

1. INTRODUCTION

Several active fields of research are focused on the form and function of collective behavior as it pertains to the acoustic environment which include biomusicology, music perception, and sensorimotor synchronization. While synchronization in music performance is evidence of our own dynamic ability for complex forms of mutual-entrainment, several groups of animal and insect populations have been shown to resort to similar modes of synchronization in the form of chorusing, coordinated signaling, and stridulation often performed as a means for reproductive practices [1] [2] [3]. While not an explicitly auditory phenomenon, the prominent paradigm of firefly signaling is a classic example that illustrates the extent to which locally interactive behavior can result in emergent, global patterns [4]. In urban and metropolitan life, the sonic world is awash with noisy, periodic sounds such as public transportation, the harsh, repetitive sounds of a construction site, or the nervous energy of restless legs under a desk. Navigating, interacting with, and understanding our physical environment oftentimes entails using auditory cues in order to guide our behavior [5] [6]. As in music, we take note of the temporal relationships between regularly occurring events in order to infer information about its source and make predictions about the future [7] [8]. However what qualities of the sounding environment allow us to infer that they are indeed connected?

In general, when events occur with enough regularity and are spaced close enough in time, we tend to attribute some coupling mechanism or process that binds them together [9] [10]. However, this becomes more complicated when we are tasked with unpacking these dynamics in the context of simultaneous sound sources that may not be coupled in simple one-to-one relationships and through which synchrony emerges over time. These systems are said to undergo a (number of) phase transitions that form a continuum of synchronous states from asynchrony to complete synchronization. Here I define a process called *constructive accumulation* that aims to describe any time-based, sonic environment or process that sees the gradual aggregation, or accumulation, of seemingly disparate sound events converging over time into a temporal order or pattern. This broad description is applicable to stridulating cricket population, groups of mechanically coupled metronomes, members of an orchestra, and the applause that follows a performance of the latter, all examples that have been modelled using synchronization strategies discussed in this paper [11] [12] [13]. It's worthwhile to take a moment and try to unpack what is meant by 'order' or 'pattern'. Since this type of behavior is prevalent in specific biological populations, there is research attempting to delineate a taxonomy of signal complexity as they relate to different types of rhythmic patterns [14]. In the present paper, 'order' or 'pattern' refers to the perception and outcome of obtaining periodically, organized sounds. Here of course we are dealing with human perception, the attribution of subjective criteria over phenomena that self-organizes in some way over time. As such, I will examine three models of collective behavior in order to further characterize such constructively accumulated processes.

2. MATHEMATICAL DESCRIPTION

2.1 Pulse-Coupled Oscillators

Pulse-coupled oscillators have been used to describe a number of mutually synchronous systems in the biological world (pacemaker cells, cricket chirping, insulin secretion in pancreas, menstrual cycles) [15] [16] [17]. Pulse-coupling often relies on a coupling function that oversees how coupling is applied when an oscillator 'fires' or 'triggers'. Phase response curves (PRC) govern this coupling relationship, parameterized by a coupling coefficient, as different PRCs shapes result in different dynamics. Here I define one of the simplest types of pulse-coupling among 'integrate-and-fire' oscillators that were initially inspired by the self aligning nature in pacemaker cells [18].

Using the model as defined Mirollo and Strogatz, a system of N pulse-coupled, integrate-and-fire (IF) oscillators, individual oscillators dynamics is defined in Equation (1), (1.1), and (1.2) [19].

$$\dot{x_i}(t) = F(x_i(t)) \quad (1)$$

$$\dot{x_i}(t^+) = 0, \text{ if } x_i(t) = 1 \quad (1.1)$$

$$x_i(t) = min(1, x_i(t^-) + m\epsilon) \quad (1.2)$$

The set of $(x_i)^N$ are the state variables in a system of N integrate-and-fire oscillators. Equation (1.1) shows how the state variable is reset upon reaching the threshold of 1 and Equation (1.2) shows how coupling coefficient, ϵ, is distributed to other oscillators upon firing. Assuming the i^{th} oscillator has not yet crossed the threshold, m is number of simultaneous pulses received and therefore the strength of its advancement dependent on the number of oscillators firing.

Equation (2) shows how the state variable, x, is related to a phase variable, θ, which simplifies the analysis such that we can examine the system in terms of the phase variable and a function that relates θ to x.

$$x_i = f(\theta_i), f : [0, 1]- > [0, 1] \quad (2)$$

Equation (3) and (4) illustrate the function, $F(x)$ and $f(\theta)$, in terms of the state variable, x, and the phase state variable θ.

$$F(x) = \frac{e^b - 1}{b} e^{-bx}, b > 0 \quad (3)$$

$$F(\theta) = \frac{1}{b} ln(1 + (e^b - 1)\theta) \, , b > 0 \quad (4)$$

Now, we can a define a new function, f, that maps the phase state variable, θ, and following the convention of Mirollo and Strogatz is defined to be monotonically increasing and concave down. Equation (1.1) shows how the oscillator is coupled to the others from the coefficient, ϵ. b is a parameter that defines the relative concavity (down) of the function, f.

More simply, each oscillator rises toward a threshold value of 1, upon which it 'fires' and is reset to 0. Upon this firing, other oscillators' phase states are either advanced by an amount, depending on the strength of ϵ, and/or pulled into the threshold for firing, whichever is less since (1.2) is an argmin function. The oscillators are globally coupled (fully connected) and maintain identical dynamics insofar as their phase state variable is governed by a function that controls its time-course, increasing monotonically (concave down) toward the threshold. The function constrains the phase state: it increases the phase state to 1 whereby it is then reset to 0.

2.1.1 Dynamics

IF pulse-coupled oscillators has been shown to eventually lead to full synchrony regardless of N and for all initial conditions and natural period if coupling is applied. Estimates for the rate of synchrony was shown to be inversely proportional to the product ϵb [19]. There is evidence to suggest that they synchronize at times proportional to the log of the number of oscillators in the network. These outcomes were a result of global, all-to-all coupling; if we allow for coupling to only occur 'locally', for example in a chain, the time needed for synchronization decreases considerably, becoming a nearly linear function of the natural period and the coupling strength [20]. Despite the longer time scale when the natural period is small and when N is large, synchrony is still inevitable and the unusual synchronizing dynamics during this build-up period are still a subject of active research.

If we allow the natural frequencies of the oscillators to take on different values, synchrony is still achievable if they are kept within bounds . Most research in these systems have looked at the kinetics of two oscillator systems and then tried to extrapolate to larger N systems, the latter often being carried out through numerical simulation: a number of papers have demonstrated how frequency mismatch and delays can lead to unusual dynamical regimes, some of which contain resonances in polyrhythmic relationships to its intrinsic frequency distributions [21]. As will be shown in a later section, these types of oscillators settle into synchrony at a much more progressive, gradual rate than the spontaneous synchronization associated with continuously coupled systems. This has a number of desirable characteristics in terms of potential for sonic generation.

2.1.2 "Scrambler" Oscillators

Networks of scrambler oscillators probably are the most basic form of a loosely coupled oscillator system from which synchrony emerges through mean-field approximation [22]. A group of scrambler oscillators of size N are comprised of pulse-coupled oscillators each containing a phasor state variable, ϕ, that increases linearly and identically from a baseline of 0 to a threshold value of 1. Upon reaching the threshold value, the oscillator fires and performs three actions: 1) "scrambles", all other oscillators (and other extant

```
for i in iterations do
    for n in N do
        osc[n].state = osc[n].state + T
        if osc[n].state ≥ 1 then
            for j in N do
                if osc[j].state ≥1–(osc[n].cluster.size/N) then
                    osc[j].cluster = osc[n].cluster
                else
                    if osc[j].state ≥ (1 − (1/N)) then
                        osc[j].cluster = increment cluster counter
            osc[n].state = 0
        else
            osc[n].state = random(0,1)
```

Table 1. Pseudocode for scrambler oscillation. NB: osc[n].cluster.size refers to the number of clusters that are tagged with the same counter number, not the size of the array itself.

'clustered' oscillators) to a new random state. Oscillators already contained within a 'cluster' get reset to the same state. 2) oscillators with states within the threshold and 1/N of the threshold are subsumed, or absorbed, into the firing oscillators cluster. 3) the oscillator that fired (and any oscillator that just clustered along with it) is reset to the baseline state of 0. Lastly, if a cluster of oscillators reaches the threshold, then the condition for absorbing other oscillators becomes (threshold – j/N) where j is the number of clusters in that group. This has the effect of allowing more oscillators to become absorbed a single cluster grows in size.

Using the description initially defined by O'Keefe et al. (2015), I provide a brief pseudocode (see Table 1) rather than a formal analytical description since this type of system is better described as an iterative process. All of the N oscillators are initialized with a different integer representing their cluster state (to group oscillators into cluster groups once they've been absorbed into a cluster) and a different randomly initialized phase state. T is the increment that sets the natural period of oscillation.

2.1.3 Dynamics

O'Keefe et al. define a natural disorder parameter that provides a measure of system's 'fragmentation' based on how many clusters are present at any given time [22]. The system starts out maximally fragmented because each oscillator is only assigned to its own cluster group. The authors derive an expression for the rate equation that illustrates how the fragmentation decreases exponentially over time (with the natural disorder parameter reaching 1/N) as more oscillators are absorbed to synchrony. In general, scrambler oscillators achieve synchrony in a much less 'productive' way as compared to simple IF coupled oscillators because oscillators outside of threshold are simply reset to random values rather than being pulled up or down according to a PRC. Oscillators are pulled into synchrony without maintaining ordering since they are reset to a random value, unlike in pulse-coupling.

2.2 Summary Statistics: Phase Coherence

For any collection of phase states, we can also look at other summary statistics that provide a useful indication of the group's synchrony. These values are the phase coherence, R, also known as a circular mean vector and the average angle of the phase coherence, ψ. Mapping each phase state (0-1) of the oscillators above onto a circle (0-2π), we can derive an expression that relates the relative 'spread' or dispersion of the swarm of phases of each oscillator to a number between 0 and 1 with larger numbers associated with more synchrony. We define the phase coherence in Equation (5) where j is a complex number.

$$Re^{j\psi} = \frac{1}{N}\sum_{i=1}^{N} e^{j\phi_i} \tag{5}$$

Figure 1 shows plots from a numerical simulation of a group of 100 scrambler oscillators over a duration of around twelve seconds.

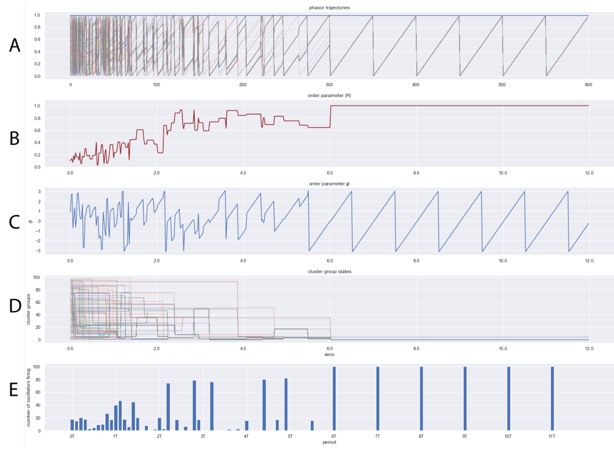

Figure 1. 100 Scrambler Oscillator Numerical Simulation. (Top to bottom) Individual oscillator phasor trajectories, Phase Coherence ($R(t)$), Average Angle ($\psi(t)$), Cluster Groupings, and Number of oscillators firing per natural period.

These plots illustrate the emergence of self-organization as the oscillators phases begin to align over a period of around six seconds. Plot a) shows the phasor trajectory of each of oscillator, Plot b) the phase coherence, $R(t)$, Plot c) the average angle, $\psi(t)$, and Plot d) the state variable of cluster groups for each oscillator over time. As more oscillators are recruited into synchrony, the number of distinct cluster groups decreases until every oscillators is in the same cluster. Lastly, Plot e) shows the number oscillators firing plotted in multiples of the natural period of the oscillators (as binned into one of ten temporal regions per natural period). Since each oscillator is identical, containing the same natural frequency, the entraining tempo that the oscillators will give rise to sync is predetermined by this initialization. Each plot provides useful visual information that allows us to gain insight into how the system self-organizes over time.

2.3 Continuous Coupling: Kuramoto Oscillators

My previous work has looked at Kuramoto oscillators from a number of different perspectives relevant for sound synthesis and music generation (for more information, please see [23] [24] [25]. Kuramoto oscillators are a type of limit-cycle oscillators with natural frequencies, ω_i, and a coupling term that continually adjusts their phases according to a sinusoidal phase response curve [26]. The natural frequencies are typically drawn from different statistical distributions and since coupling is applied at all times, synchrony can result if coupling surpasses a critical coupling value. The governing equation for a group of N Kuramoto oscillators is shown in Equation (6).

$$\dot{\phi_{\mathrm{i}}} = \omega_i + \frac{K_i}{N} \sum_{j!=i}^{N} sin(\phi_j - \phi_i) \quad (6)$$

Using the phase coherence from Equation (5), we can rewrite Equation (6) in terms of the complex order parameters, R and psi as shown in Equation (7) where an extra component, $\Lambda_e(\phi_i)$, is also added to allow for external forcing.

$$\dot{\phi_{\mathrm{i}}} = \omega_i + \Lambda_e(\phi_i) + \frac{K_i}{N} R \sum_{j=1}^{N} sin(\psi - \phi_i) \quad (7)$$

2.3.1 Dynamics

While explicit coupling is a function of the phase coherence, the external forcing function has been shown to modulate the time-varying mean field and thereby modulate the phase coherence over time which allows us to effectively 'tune' the synchrony of the system [27]. In this orientation, each oscillator interacts only with the mean-field approximated complex order parameter values, R and ψ. It's useful to imagine R and ψ in polar form where it's visually represented as a time-varying phasor, R(t)$\underline{/\psi(t)^{\circ}}$, moving about a circle. $\psi(t)$ traces out the center of mass of the swarm of points, each point representing an oscillator, and $R(t)$ increases in length when the oscillators' phases are more aligned. The external forcing function, $\Lambda_e(\phi_i)$ modifies the the natural frequency distribution and allows for different modes of both forced and mutual entrainment [28].

If we let $g(\omega)$ be a normal distribution from which the oscillators' natural frequencies are drawn and allow to N go to ∞, the critical coupling–the value, K_c, for which the synchrony emerges–has been shown to be a function of the mean natural frequency, ω_c, found in the distribution ($K_c = 2/(\pi g(\omega_c))$. At the critical coupling, synchrony emerges very rapidly and spontaneously, often referred to as a first-order phase transition which associated with an abrupt jump from a low value or R to a large one.

Allowing the parameters, N, ω_i, K_i, and $\Lambda_e(\phi_i)$, to take on time varying values allows for a range of unusual semi-synchronous states to emerge including partial states of synchronization, frequency locking, chimeric states, bellepherone states (multiple coherent clusters), and resonant states [29] [30] [21].

3. RHYTHMIC GENERATION SCHEMES

3.1 Oscillator Phase Mapping to Control Signals

The collective, self-organizing behaviors exhibited by these systems can be meaningfully applied to sound in a number of ways and similar topics have been explored in computer music research [31] [32]. Through relatively simple parameterization, we can exploit the system dynamics in such a way to produce a range of sonic outcomes that characterize constructive accumulation. This involves treating the dynamical systems as generative models that map the system output states to control signals that can be applied to sound production.

Through numerical simulation, we can plan, control, and compose for different outcomes depending on our musical intentions. Constructive accumulation deals specifically with the temporal organization of sound onsets; that is we are concerned with the building up of sonic mass using small, discrete sonic objects, a process that shares similarities in many approaches to computer music such as concatenative and granular synthesis, microsound, and compositional techniques involved in micropolyphony [33], [34], [35].

Circle maps are widely used in mathematics and physics to depict iterative phase states and have been employed in the service of sound synthesis in several ways [36] [37]. Figure 3 illustrates such a circle map with several oscillators represented as black points, each moving about the circle with a frequency, $\omega_i = \dot{\phi_i}$. These control signals can subsequently be mapped to any number of musical parameters. The most simple form of this is to apply thresholds along the phase space and trigger a sound event once per cycle, when $\phi_i = 0$. However, extending this notion we can trigger multiple sound onsets during the course of a single limit-cycle to create a number of different rhythmic patterns. More formally stated: if we let, θ_m, be M number of points along the unit circle, we create triggers using pulses when $\phi_i == \theta_m$.

3.1.1 Using mean field parameters

Similarly, we can use the complex order parameters, the phase coherence, $R(t)$, and the average angle, $\psi(t)$ as another control signal from which to assign to other control triggers. In this case, we can use the magnitude of the phase coherence and the angle of $\psi(t)$ trigger events based on a similar collection of m thresholded values defined as r_{tm} and ψ_{tm}. In this way, other control signals are only generated during specific regions of the dynamic regime of the system. The Figure shows two phase coherence magnitude trigger lengths ($r_{t1,t2}$), and two phase coherence angle trigger angles ($\psi_{t1,t2}$). Similarly, we allow the set of trigger points to take on different values over time in order to introduce more rhythmic variation into the system.

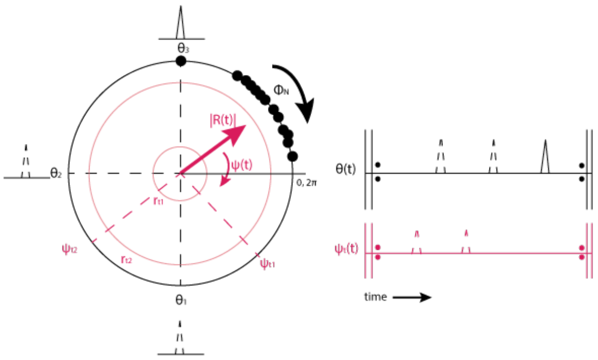

Figure 2. Circle map illustration of oscillator phase and complex order mapping to control signal triggers.

4. PSYCHOACOUSTIC CONSIDERATIONS: TIMBRE VS. RHYTHM FORMATION

Our ability to discern, process, and entrain to multiple, simultaneous sounds has been looked at from a number of perspectives in music cognition, psychoacoustics, and auditory scene analysis. Research has shown that auditory stream formation depends largely on the temporal coherence inherent in a sound signal [38]. While much of this line of research focuses on the separation and identification of simultaneous sound sources ('the cocktail party' problem'), the dynamical systems and generation scheme thus described are concerned with concurrent, quasi-periodic rhythmic sources, an auditory scene that has been less studied in sensorimotor synchronization studies [9]. More specifically, how does the simultaneous presentation of multiple, yet identical, periodic sound sources differ from attending to a normal acoustic environment that contains an abundance of non-coupled sounds?

Studies in sound texture perception provide one clue into how the brain encodes such auditory scenes comprised of similar acoustic events. McDermott and Simoncelli were able to realistically approximate soundscapes of relative temporal homogeneity (e.g. insect swarms, running water, and applause) by recreating the time-averaged statistics of an audio signal's frequency decomposition [39]. Since attending to every singular sources in such a texture would be cognitively demanding, this study supports the idea that when confronted with a large number of collective sound sources, we adapt to a temporal averaging strategy whereby timbre is identified through statistical inference. However, applying coupling to these systems, the resultant signals are no longer statistically 'stationary' and the emergence of cognitive processes implicated in beat extraction most likely take precedence. For example, one study looked at how 'embedded', repeated temporal-spectral acoustic features in mixtures have been shown to facilitate discernment and source identification [40].

Similarly, research in auditory scene analysis provides other clues. For instance, gestalt grouping and stream convergence based on temporal proximity has been shown to be a factor in how we perceptually merge different sounds with differing rhythms [41] [42]. If enough onsets are temporally proximate, a signal's temporal envelope also provides auditory cues that allow for the emergent rhythm to be detected if the constituent elements are of similar, homogenous textures [43]. In developing his 'sound mass' pieces in the mid 20th century, the composer György Ligeti remarked how sounds produced around 18 times/second create the perception of a continuous texture, an observation also noted in Iannis Xenakis' writings [35]. In his music, global rhythms emerge from the local interactions of independent instruments, a compositional technique defined as a sort of micro-polyphony.

One unusual auditory environment that produces a similar spectrum of textures is the phenomena of synchronized clapping that occurs during audience applause. Several studies have examined the crowd dynamics using systems of coupled oscillators where they proposed that an audience will instinctually adapt their natural clapping frequency to a lower frequency which has the effect of reducing the frequency dispersion of intrinsic frequencies within the group [11] [44]. In this self-organizing behavior, audience members are most likely making phase adjustments to synchronize with other audience members in their spatial vicinity.

My previous experimental research has looked at how we entrain to networks of coupled oscillators by examining how beat extraction is carried out when confronted with sounds generated in a similar schema [45]. In one study, participants were asked to tap to the 'beat' to auditory stimuli where coupling strengths were modified from 'weak' to 'strong'. Participants displayed a wide range of tap strategies that we classified as isochronous, patterned (non-isochronous but rhythmic), or dense (rapid tapping) with the latter two strategies associated more with weaker coupled sounds. This supports the notion that such auditory scenes encapsulate different understandings of sound through listening and interaction, perhaps through a form of active sensing. We are particularly interested in the quasi-periodic states that arise just before synchronization and at what point are we able to latch onto the emergent pattern in the form of an entraining rhythm. This is illustrated in Figure 1 during the first half of the sequence where there begins a salient formation of 'lumps' of oscillator firings around an increasingly narrower and periodic temporal centers.

These research questions motivate the need to carry out alternative approaches for reproducing these dynamics in artistic and aesthetically mediated environments. With this in mind, I present one such instantiation of these ideas in my installation, 'In Praise of Idleness'.

5. IN PRAISE OF IDLENESS

5.1 About the Piece

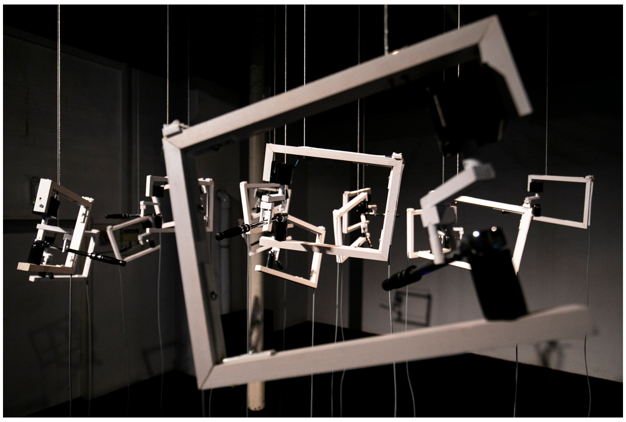

Figure 3. View of the Installation

In this site-specific installation [1] that premiered at Galerija Alkatraz in Ljubjlana, several large socket wrenches (“ratchets”) are driven by an assembly of motors that kinetically rotate them in a number of coordinated ways. Mechanically, ratchets have an internal clicking mechanism that produces sound when it is rotated. As an audio-visual symbol linking mechanical labour with the passage of time, the socket wrenches evoke the ambient landscapes of swarms of stridulating insects, metronomes, or ticking clocks. In order to control their behavior, several different algorithmic techniques are used to choreograph the socket wrenches’ collective dynamics, the result being a cooperative and sometimes crude synchronisation of movement and sound that is realized through their kinetic design and through the “sound(s) of their own making”, a reference to Robert Morris’s piece of a similar title. Dialoguing with Jacques Attali’s notion of noise as constituting “a political economy of music”–one that is intimately tied up with society’s modes of production–this piece points to the ways in which simple repetition can be exploited to build up complex auditory textures through the aforementioned process of constructive accumulation [46]. The title, "In Praise of Idleness", is taken from an essay by Bertrand Russel in which he calls into question our cultural obsession with work as inherently virtuous.

5.2 Technical Requirements

The installation uses the rotational motion of 19 socket ratchets as a way to induce small mechanical clicks that serve as sound events using the circle map scheme shown in Figure 3. Each ratchet is housed in a wooden, c-shaped form with is hung from the ceiling. A small lever arm, attached to the shaft of a NEMA 17 stepper motor (0.59 N-m, 1.7A, 1.8°), rotates the ratchet when a pulsed voltage is applied to the stepper coils. Each stepper motor is controlled with a dedicated stepper motor driver (TBB6600) and powered with an individual power supply (12 V, 2 A). The required torque to drive the ratchet lever was calculated to be around 0.24 N-m.

An Arduino mega was used to control the rotation of the stepper motors using the Stepper Library and all the algorithms were written using the Arduino language. Numerical integration for the IF pulse-coupled, and Kuramoto oscillators using a forward Euler integration scheme which has been shown to be an adequate approximation when the increment step size is small with respect to the intrinsic period [47] [48]. The motor drivers are provided with a 5V PWM signal from the arduino outputs that rotate the ratchets to produce a single click or a "stridulation" of clicks which interestingly shares a similar kinetic mechanism with the plectrum of a cricket body [2].

5.3 Composition

Since rich complexity exhibited by collective systems is often a function of large-scale statistical processes, the installation was designed to bring out a dynamic range of collective behaviors. Given their sensitive nature of the parameters spaces, these systems are not conducive for meticulously planned approaches for deterministic outcomes. Rather, using the knowledge imparted from their dynamics, they encourage the careful design of constraints that will ultimately yield approximate results. The different processes associated with constructive accumulation were mainly used to evoke the presence of a seemingly autonomous and aware swarm of sound. Different outcomes are realized by modulating parameters associated with each of the collective behaviors.

5.3.1 Composition of Time points

The installation features different behavioral modes that are based on states associated with the coupled oscillator regimes above. These modes are written as routines in the code such that when they are triggered, the relevant parameters and time points are initialized and the generative model is allowed to play out. The routines themselves are randomly selected.

The parameters for each of the coupled oscillator algorithms were made time-varying by setting target values at discrete time points and then either stepping or linearly interpolated between values. Numerical simulation were performed in Python to approximate the output behavior of the system. For demonstration purposes, an example of six of the oscillators’ time points in graph form is illustrated in Figure 4 which shows the time varying coupling coefficients (K_i) and the natural frequency dispersion, (σ_ω) overlayed with the center natural frequency, (ω_c). For this scenario with continuous Kuramoto oscillators, there is no coupling at the beginning with each oscillator simply moving at its natural frequency as dictated by natural frequencies chosen by the distribution with ω_c with variance σ_ω.

[1] Please see: `https://www.youtube.com/watch?v=68sq2jm_WCw&t=0s`

The bottom Figure shows 5 different numerical simulations of the phase coherence magnitude, $|R(t)|$. In particular demonstrates how sensitive the system is to initial conditions and changes in the frequency distribution. Time points were also created that allow oscillators to be enabled (turned on/off).

I provide a few descriptions of the ways in which each of the coupling processes are used in the piece and provide a timestamp in the documentation video (see footnote above) where this behavior occurs.

- Pulse-Coupling: Simple synchronization over different time periods as parameterized increasing coupling strength, ϵ, over time. Constructive accumulation tends to happen more progressively as compared to the other methods. (0:00 to 0:41).
- Scrambler oscillators: Synchronization with a relatively slow natural period (2:10 - 2:30).
- Kuramoto Oscillation: Synchronization as applied to rotational speed rather than pulses to produce a noisy timbral texture (00:52 - 01:09). Partial desynchronization after full synchrony (1:10 - 1:22). Partial synchronization by applying feedback to K_i using phase coherence state, R (1:30 - 2:10).

Other coordinative behaviors were also implemented in the code which are not discussed here. This includes routines for setting the speed and rotation duration for each ratchet without coupling and a simple time based triggers that pulses the ratchets at strict periods.

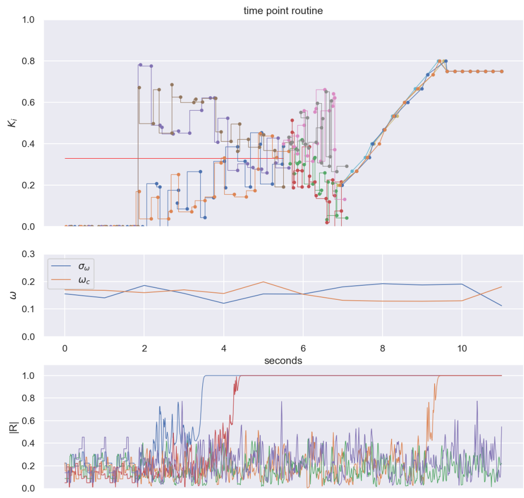

Figure 4. Timepoint plot for a single instance of a Kuramoto Routine. The top plot shows the coupling coefficients as a function of time for six of the nineteen oscillators. The red line indicates the critical coupling value. The middle plot shows the timepoints for the center frequency, ω_c, and the variance, σ_c, of the intrinsic frequencies of the oscillators. The bottom plot shows the phase coherence, R, over 5 numerical simulations. This demonstrates different behavioral outcomes that largely depend on the intrinsic frequency distribution.

6. CONCLUSIONS

Ultimately, this artwork demonstrates one such artistic application of the concept of constructive accumulation as a way to explore different form of synchronization as a sensory phenomena. Using outcomes of research looking into these systems' dynamics, this paper attempted to summarize these processes from the perspective of such a process where sound onsets coalesce in time from disorder to emergent rhythm. By presenting relevant research in auditory scene analysis and beat extraction/entrainment, I hoped to suggest several approaches to characterizing the auditory reception of such sensory phenomena and how they might encourage different listening modes and strategies. With an abundance of synchronization strategies being actively examined in this domain, future work would do well to look at more recent developments that may be particularly suitable for sonic engagement. One such area concerns Kuramoto models related to higher dimensional spaces that have shown great promise for reproducing complex behaviors associated with swarms and flocking [29] [49].

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